Matrices

Kronecker Delta	$\delta_{ij} = 1$ If i = j $\delta_{ij} = 0$ Otherwise
Product	Given A (m x n) and B (n x o): $[AB]_{ij} = a_{ik}b_{kj} = A_{row(i)}.B_{col(j)}$
Trace	$Tr(A) = \sum_i a_{ii}$, $Tr(AB) = Tr(BA)$
Transpose	$[A^T]_{ij} = [A]_{ji}$, $(AB)^T = B^T A^T$
Symmetry	$A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T)$
Quadratic	$x^T Ax = 1$, x column, A symmetric
Determinants	$\det(A)\det(B) = \det(AB)$ $\det(\lambda A) = \lambda^{\text{dimension}} \det(A)$ $\det(A^T) = \det(A)$ $\det(A^{-1}) = \frac{1}{\det(A)}$ $\det(A) = -\det(A')$ (row interchange) $\det(A) = 0$ (given identical rows) $\det(A) = \det(A')$ (row combination)
Inverse	$[A^{-1}]_{jk} = \frac{1}{\det(A)}\Delta_{kj}$, $(AB)^{-1} = B^{-1}A^{-1}$ $\Delta_{jk} = (-1)^{j+k} M_{jk}$, M = minor
Cramer's Rule	$x_i = \frac{\det(A_i)}{\det(A)}$ given Ax=b
Orthogonal	$AA^T = I$, $\det(A) = \pm 1$, rows of A are orthonormal
Rotation	Orthogonal with det(A) = 1, e.g.: $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$
Eigenvalues	$Ax = \lambda x$, $\det(A - I\lambda) = 0$ $P = \begin{bmatrix} e_1 \vert & e_2 \end{bmatrix}$, $P^T AP = \Lambda = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$ Since $x^T Ax = 1$, $y = P^T x$ =best axis

Fourier Series

Orthogonal over [a, b]	$\int_b^a f(x)g(x) = 0$ if f, g orthogonal
Fourier Expansion	$f(x) = \frac{1}{2}a_0 + \sum_{n=0}^{\infty}(a_n \cos(\frac{n\pi x}{L}) + b_n \sin(\frac{n\pi x}{L}))$ $a_0 = \frac{1}{L}\int_{-L}^{L} f(x)dx$ $a_m = \frac{1}{L}\int_{-L}^{L} \cos(\frac{m\pi x}{L}) f(x)dx$ $b_m = \frac{1}{L}\int_{-L}^{L} \sin(\frac{m\pi x}{L}) f(x)dx$
Observations	For odd functions, only sin terms appear At discontinuities in f(x), the Fourier approximation takes the mean of the value of the discontinuity when approached from different directions (called the Gibbs phenomenon)
Calculus	Always correct to integrate Correct to differentiate if its Fourier coefficients tend to 0 at least as fast as n^{-2}, n tends to ∞
Incident Power	$P = \int_{-L}^{L} (f(x))^2 dx$
Parseval's Theorem	$\int_{-L}^{L} (f(x))^2 dx = L(\frac{1}{2}a_0^2 + \sum_{n=1}^{\infty}(a_n^2 + b_n^2))$
Power Spectrum	$\{a_0^2, a_1^2 + b_1^2, .., a_m^2 + b_m^2, ..\}$
Half Range Series	Can extend the function either in an odd or even fashion with all sin or cos terms respectively