

## Probability

Union  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Conditional  $P(F | E) = \frac{P(F \cap E)}{P(E)}$

Permutation  ${}^n P_r = \frac{n!}{(n-r)!}$ ,  ${}^n C_r = \frac{n!}{(n-r)!r!}$

Mean  $\langle x \rangle = \frac{\sum n_i x_i}{N} = \sum P_i x_i$

Variance  $\sigma^2 = \langle (x - \langle x \rangle)^2 \rangle = \langle x^2 \rangle - \langle x \rangle^2$

Binomial  $P(r) = {}^n C_r p^r (1-p)^{n-r}$   
 $\langle r \rangle = np$ ,  $\sigma_r^2 = np(1-p)$

Cumulative Probability  $F(x) = P(X \leq x)$   
 $f(x) = \frac{dF}{dx}$ ,  $\int f(x) dx = 1$

Gaussian  $f(t) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(t-\mu)^2}$   
 $\langle t \rangle = \mu$ ,  $\sigma_t^2 = \sigma^2$

## Partial Derivatives

Basics  $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$ ,  $\nabla f = (\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \dots)$

Exactness  $df = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy$ ,  $\frac{df}{dt} = \frac{\partial f}{\partial x} \frac{dx}{dt} + \frac{\partial f}{\partial y} \frac{dy}{dt}$

Chain Rule  $\frac{\partial f}{\partial x} = \frac{\partial f}{\partial t} \frac{\partial t}{\partial x}$

Maxima  $f_{xx} > 0$ ,  $f_{yy} > 0$ ,  $f_{xx} f_{yy} > f_{xy}^2$

Minima  $f_{xx} < 0$ ,  $f_{yy} < 0$ ,  $f_{xx} f_{yy} > f_{xy}^2$

## Fields And Gradients

Types  $\phi = \phi(x, y, z)$ ,  $\underline{V} = \underline{V}(x, y, z) = \nabla \phi$

Geometry  $\phi(\underline{x} + d\underline{x}) - \phi(\underline{x}) = \nabla \phi \cdot d\underline{x}$   
 $\nabla \phi$  is the normal to a plane of constant  $\phi$

Line  $\int_{\Gamma} \underline{F}(\underline{x}(t)) \cdot d\underline{x} = \int_{\Gamma} (\underline{F}(\underline{x}(t)) \cdot \frac{d\underline{x}}{dt}) dt$

Conservatism For any closed curve  $\int_A^A \underline{F} \cdot d\underline{x} = 0$

This is true for any  $\underline{F} = \nabla \Phi$

Surface  $S = \phi(x, y, z) = C \Rightarrow \underline{n} = \nabla \phi$ ,  $d\underline{A} = \widehat{\underline{n}} dA$

$$\int \underline{F} \cdot d\underline{S} = \int (\underline{F}(\underline{x} \text{ restricted to } S) \cdot \underline{n}) dS$$

$$dS = r^2 \sin \theta d\theta d\phi$$

Divergence  $\text{div } \underline{F} = \nabla \cdot \underline{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$

Div. Theorem  $\int_D \text{div } \underline{F} dV = \int_S \underline{F} \cdot d\underline{S}$  for S bounded D

Curl  $\text{curl } \underline{F} = \nabla \wedge \underline{F} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix}$

Curl Theorem  $\text{curl } \underline{F} = 0 \Leftrightarrow \underline{F} = \nabla \phi$

Stokes Theorem  $\int_S \text{curl } \underline{F} \cdot d\underline{S} = \int_L \underline{F} \cdot d\underline{l}$  for L bounded S